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Mathematical Modeling of Seismomagnetic Effects Arising in the Seismic Wave Motion in the Earth's Constant Magnetic Field

B. G. Mikhailenko and O. N. Soboleva
Computing Center
Russian Academy of Science
630 090 Novosibirsk, Russia

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Abstract—The induction seismomagnetic effects arising in the seismic wave motion in the constant Earth's magnetic field are numerically studied in this article. The phenomenon is described as a simultaneous solution of the system of elastic equations and quasi-stationary Maxwell's equations with displacement velocity components. For solving the problem, we use numerical-analytical algorithm based on the finite Fourie transform. The obtained system of ordinary differential equations is solved by the factorization method.

Keywords—Seismomagnetic effects, Numerical methods for solving the system of elastic equations, Quasi-stationary Maxwell's equations.

1. INTRODUCTION

It is known that when a conductive wire frame is moving in a constant magnetic field, an electric current and a variable magnetic field are generated in the frame.

Similar phenomena can be observed when a seismic wave propagates in the Earth's constant magnetic field. The seismic wave, with its forward and back wave fronts is analogous to the wire frame. A conductive medium between the forward and the back wave vibrates in the Earth's constant magnetic field, which brings about local geomagnetic variations. Local geomagnetic variations, propagating simultaneously with the seismic wave diffusing into the medium are termed seismomagnetic waves. These waves contain information about both electromagnetic and elastic parameters of a medium.

The electromagnetic wave rides the "back" of the seismic wave, that is, the induced electromagnetic wave is "frozen" into the seismic wave and propagates either with P- or with S-seismic wave velocity, depending on the type of waves. The dominant frequency and the velocity of the induced seismomagnetic wave is equal to the frequency and velocity of the seismic wave.

Lately, some attempts have been made to measure and implement the electrical currents generated by seismic waves and to develop methods of electroseismic prospecting [1]. The electroseismic approach is different from the seismomagnetic method. Whereas the electroseismic effect results from a local effect of the seismic wave interactions with the interface between elastic media and shallow layers of subsurface fluids, the seismomagnetic effect is based on the interaction of seismic

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waves with the Earth's magnetic field. The result of such an interaction is the induced electromagnetic wave which propagates with the seismic wave, but not with the light speed, as in the case of the electroseismic effect.

The results (see [2,3]) indicate to the simultaneous propagation of the seismic wave together with the induced geomagnetic variation, and to the fact that it is possible to record the geomagnetic variation.

2. MATHEMATICAL MODEL FOR SEISMOMAGNETIC EFFECT

The phenomenon of the seismomagnetic effect caused by an explosion in an elastic medium is described as a simultaneous solution of the self-consistent system of elastic equations with the Lorenz force and quasi-stationary Maxwell's equations with displacement velocity components. The first paper in this area was by Knopoff [4], who concluded that the Earth's magnetic field weakly affects the wave propagation velocity; however, Knopoff did not study seismomagnetic waves. The theory of the interaction of elastic and magnetic fields proposed by Knopoff was further developed in the solid state physics. At present, a new 1 area of research of the magnetoelasticity theory is gaining more importance [5]. We use the system of equations derived by Novacki

$$V_p^2 \operatorname{grad}\operatorname{div}\boldsymbol{U} - V_s^2 \operatorname{rot}\operatorname{rot}\boldsymbol{U} = \frac{\partial^2 \boldsymbol{U}}{\partial t^2} + \boldsymbol{F}, \tag{1}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \operatorname{rot}\left(\frac{\partial \boldsymbol{U}}{\partial t} \times \boldsymbol{H}_0\right) + \beta \Delta \boldsymbol{h}, \tag{2}$$

$$\operatorname{div}\boldsymbol{h} = 0. \tag{3}$$

For the vacuum $z < 0$,

$$\Delta \boldsymbol{h} = 0. \tag{4}$$

Components of the seismoelectric field can be found from the following equations:

$$\boldsymbol{E} = \sigma^{-1}\operatorname{rot}\boldsymbol{h} - \mu_0 \frac{\partial \boldsymbol{U}}{\partial t} \times \boldsymbol{H}_0, \tag{5}$$

for the vacuum $z < 0$:

$$\operatorname{rot}\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t}. \tag{6}$$

Here $\boldsymbol{U}$ is the displacement vector of the elastic medium, and $\boldsymbol{H}_0$ is the strength vector of the Earth's magnetic field.

Local geomagnetic variations of the magnetic field induced into the medium due to the seismic wave propagation can be represented as $\boldsymbol{H}_0 \pm \boldsymbol{h}$, where $\boldsymbol{h}$ is assumed to be small as compared to $\boldsymbol{H}_0$. Velocities of longitudinal V_p- and transverse V_s-waves, density ρ, and conductivity σ are assumed to be constant within each layer. Magnetic permeability μ_0 is assumed constant for all the layers.

The continuity conditions should be satisfied for the displacement components, strengths τ_{xz}, τ_{yz}, τ_{zz} of the elastic medium and magnetic field components, tangent components of the electrical field for the contact boundary for any two layers.

On the boundary with vacuum for $z = 0$, the strength $\tau_{xz} = 0$, $\tau_{yz} = 0$, $\tau_{zz} = 0$, magnetic field components and the normal component of the electrical field are continuous. At the infinity, movement is absent. The source of the seismic wave

$$\boldsymbol{F} = f(t)\left[M_{ij}\frac{\partial \delta\left(\boldsymbol{x} - \boldsymbol{x}_0\right)}{\partial x_j} + \boldsymbol{F}_0 \delta\left(\boldsymbol{x} - \boldsymbol{x}_0\right)\right], \tag{7}$$

where $\boldsymbol{x} = (x, y, z)$, $M_{i,j}$ are the components of seismic moment, $f(t)$ is Puzyrerv's impulse:

$$f(t) = \exp\left[-\frac{2\pi\omega_0\left(t - t_0\right)^2}{\gamma^2}\right]\sin\left(2\pi\omega_0\left(t - t_0\right)\right),$$

$\boldsymbol{F}_0$ is vector of the force.

3. NUMERICAL METHOD

To solve the system of equations (1)–(4), we will make use of the earlier developed numerical-analytical algorithms [6] employing the finite Fourier transform along the coordinates x, y, t. The obtained system of ordinary equations is the following:

$$\begin{aligned}
&\frac{d^2u_x}{dz^2} - \left(\frac{V_p^2}{V_s^2}k_x^2 + k_y^2 - \frac{\omega^2}{V_s^2}\right)u_x - k_xk_y\frac{\left(V_p^2 - V_s^2\right)}{V_s^2}u_y + ik_x\frac{\left(V_p^2 - V_s^2\right)}{V_s^2}\frac{du_z}{dz} = 0,\\
&\frac{d^2u_y}{dz^2} - k_xk_y\frac{\left(V_p^2 - V_s^2\right)}{V_s^2}u_x - \left(k_x^2 + \frac{V_p^2}{V_s^2}k_y^2 - \frac{\omega^2}{V_s^2}\right)u_y + ik_y\frac{\left(V_p^2 - V_s^2\right)}{V_s^2}\frac{du_z}{dz} = 0,\\
&\frac{d^2u_z}{dz^2} + \frac{\left(V_p^2 - V_s^2\right)}{V_p^2}\left(ik_x\frac{du_x}{dz} + ik_y\frac{du_y}{dz}\right) - \left(\frac{V_s^2}{V_p^2}k_x^2 + \frac{V_s^2}{V_p^2}k_y^2 - \frac{\omega_2}{V_p^2}\right)u_z = 0.
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\frac{d^2h_x}{dz^2} - \tau^2h_x = \frac{i\omega}{\beta}\left[H_{0x}\left(\frac{du_z}{dz} + ik_yu_y\right) - H_{0y}ik_yu_x - H_{0z}\frac{du_x}{dz}\right],\\
&\frac{d^2h_y}{dz^2} - \tau^2h_y = \frac{i\omega}{\beta}\left[-H_{0x}ik_xu_y + H_{0y}\left(\frac{du_z}{dz} + ik_xu_x\right) - H_{0z}\frac{du_y}{dz}\right],\\
&\frac{d^2h_z}{dz^2} - \tau^2h_z = \frac{i\omega}{\beta}\left[-H_{0x}ik_xu_z - H_{0y}ik_yu_z + H_{0z}\left(ik_xu_x + ik_yu_y\right)\right],
\end{aligned} \tag{9}$$

where $\tau^2 = k_x^2 + k_y^2 + i\omega/\beta$. The source $\vec{F}$ from the right-hand side of equations (10) is carried over to the point $z = z_0$ by the standard technique.

The solution of equations (11) we find in the form

$$h_x = C_{1l}^j e^{\tau_j z} + C_{2l}^j e^{-\tau_j z} + \varphi_l^j, \qquad l = x, y, z, \tag{10}$$

where j is a number of the layer. If the partial solutions φ_x, φ_y, φ_z of equations (11) are known, we can find constants C^j using the well-known (for such problems) recurrent formulas taking into account the boundary conditions and conjugation conditions for layers. It is more difficult to define the partial solution of equations (11) as the difficulties of construction of the analytical solution in every layer of the elastic theory equations by matrix methods are well known. For solving equations (10), we used the factorization method [6]. Here we introduce the potentials

$$\begin{aligned}
u_x &= ik_xw_1 + \frac{dw_2}{dz},\\
u_y &= ik_yw_1 + \frac{dw_3}{dz},\\
u_z &= -ik_xw_2 - ik_yw_3 + \frac{dw_1}{dz}.
\end{aligned}$$

Then from (10),

$$\frac{d^2w_1}{dz} = R^2w_1, \qquad \frac{d^2w_2}{dz} = S^2w_2, \qquad \frac{d^2w_3}{dz} = S^2w_3, \tag{11}$$

where $R^2 = k_x^2 + k_y^2 - w^2/V_p^2$, $S^2 = k_x^2 + k_y^2 - w^2/V_s^2$.

Following [6], we introduce a new unknown matrix of the functions A

$$\frac{d\mathbf{w}}{dz} = A\mathbf{w}. \tag{12}$$

Provided that (14) identically satisfies the equations in potentials (13), we obtain Riccati's matrix equations for determining a_{ij} in each layer. Riccati's equation admits an analytical solution in the closed form. In its final form, the algorithm is as follows. First, we perform the same procedure of calculating a_{ij} from above up to the point $z = z_0$, where the source is located, using the conjugation conditions between layers and Riccati's equations. Then this procedure is repeated from the bottom upwards. Taking into account the conditions in the source, we obtain values of potentials in each layer. If we present the prototypes of displacements in the form

$$u_l = C_{3l}^j e^{R_j z} + C_{4l}^j e^{-R_j z} + C_{5l}^j e^{S_j z} + C_{6l}^j e^{-S_j z}, \qquad l = x, y, z. \tag{13}$$

We will be able to find the constant C_{il}^j in each layer and to construct the partial solution for equations (11). Note, this approach has no computational restrictions when doing calculations with high frequencies.

4. SOME RESULTS OF NUMERICAL MODELING OF SEISMOMAGNETIC WAVE

Consider some dynamic features of seismomagnetic waves. Each kind of the seismic wave generates an electromagnetic wave associated with it and propagating with the same velocity. The electromagnetic wave generated by a seismic wave of a given kind we call the seismomagnetic wave of the same kind (e.g., Rayleigh seismomagnetic wave, longitudinal seismomagnetic wave, transverse seismomagnetic wave, etc.). As compared to the longitudinal wave the seismomagnetic wave is transverse, as any other electromagnetic wave. However, the longitudinal seismomagnetic wave propagates with longitudinal seismic wave velocity. Basic dynamic features of seismomagnetic waves for homogeneous elastic media were considered in [7]. Further, we convert components of both seismic and seismomagnetic fields from x, y, z to the spherical coordinate system for stratified elastic media. We transformed all the components into dimensionless units. We divided the components of the seismomagnetic field by the maximum amplitude of all its components and the components of seismic field by the maximum amplitude of all its components.

Figure 1.

Figure 1 shows radial, tangential components of the displacement of the elastic wave at the point $r_0 = 3\lambda$, where λ is the dominant P-wavelength in the elastic medium and the radial, tangential components of seismomagnetic field at the same point for different angles $\hat{\theta}$; $\hat{\theta}$ is the angle between the strength vector of the Earth's magnetic field $\boldsymbol{H}_0$ and the vertical axis z. The elastic model is used with an explosive point source near $z = 0$, for this case transverse components of all waves are equal to zero. Parameters of the model are the following: $V_{p1} = 1000\,\mathrm{m/s}$, $V_{p2} = 2000\,\mathrm{m/s}$, $V_{si} = V_{pi}/1.73$, $i = 1, 2$, the depth of layer $h = \lambda$, strength of geomagnetic field $H_0 = 40\,\mathrm{A/m}$, conductivity $\sigma = 0.01\,\mathrm{Cm/m}$. The figure shows that the phase and first arrivals of geomagnetic variations coincide with the analogous characteristics of the seismic waves. The first wave is the longitudinal wave P, the second wave is the Rayleigh wave, and the third wave is a wave reflected from the boundary of the layer. The radial and tangential components of P-

Figure 2.

and the Rayleigh seismomagnetic waves have the same circular polarization. The amplitude of P-seismomagnetic wave decreases with the increase of the angle $\hat{\theta}$, while the amplitude of the Rayleigh wave increases.

Figure 2 shows the radial, tangential, transverse components of elastic displacements and seismomagnetic waves. The elastic model and its parameters are the same as in Figure 1. The point source located near $z = 0$ is the source of the horizontal force type. In this case, we have radial, tangential, and transverse components of all waves. The amplitude of P-seismomagnetic wave decreases with the increase of the angle $\hat{\theta}$, while the amplitude of the Rayleigh wave increases.

REFERENCES

1. A.H. Thompson *et al.*, Electroseismic prospecting, In *Abstracts, SEG*, pp. 425–427, Houston, TX, (1991).
2. S.V. Anisimov, M.B. Gokhberg, E.A. Ivanov, M.V. Pedanov, N.N.Rusakov, V.A. Troizhkya and V.E. Goncharov, Short period oscillations of electromagnetic field of the Earth after explosion, *Dokl. Acad. Nauk SSSR* **281** (3), 556–559 (1985).
3. M.A. Sadovsky, Editor, *Electro Magnetic Precursors of Earthquakes*, Dokl. Acad., Nauka, (1980).
4. L. Knopoff, The interaction between elastic wave motions and a magnetic field in electrical conductors, *J. Geophys. Res.* **60**, 441–456 (1955).
5. W. Novacki, Electromagnetic effects in solid bodies, In *Panstwowe Wydawnictwo Naukowe*, p. 159, Warszava, (1983).
6. A.G. Fatianov and B.G. Mikhailenko, Numerically-analytical method for calculation of theoretical seismograms in layered-inhomogeneous anelastic media, In *Proceedings of the 7^{th} International Mathematical Geophysics Seminar held at the Free University of Berlin*, February 8–11, 1989.
7. B.G. Mikhailenko and O.N. Soboleva, Numerical modeling of seismomagnetic effects in an elastic medium, In *Advanced Mathematics: Computations and Applications Proceedings of the International Conference AMCA-95*, pp. 722–730, Novosibirsk, Russia, (1995).
8. M.G. Gokhberg *et al.*, Electromagnetic radiation of rocks under conditions of explosion, *Dokl. Acad. Nauk SSSR* **295** (2), 321–335 (1987).
9. M.G. Gokhberg *et al.*, Electromagnetic field of an earthquake source, *Dokl. Acad. Nauk SSSR* **235** (1), 62–65 (1990).
10. B.G. Mikhailenko, Numerical modeling of seismomagnetic effects in an elastic medium and their application in geophysical exploration, In *Abstracts, SEG*, pp. 428–431, Houston, TX, (1991).
11. K. Aki and P.G. Richards, *Quantitative seismology*, Volume 1, W.H. Freeman, San Francisco, (1980).