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Universality in solar flare, magnetic storm and earthquake dynamics using Tsallis statistical mechanics

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ABSTRACT

The universal character of the dynamics of various extreme phenomena is an outstanding scientific challenge. We show that X-ray flux and D_{st} time series during powerful solar flares and intense magnetic storms, respectively, obey a nonextensive energy distribution function for earthquake dynamics with similar values for the Tsallis entropic index q. Thus, evidence for universality in solar flares, magnetic storms and earthquakes arise naturally in the framework of Tsallis statistical mechanics. The observed similarity suggests a common approach to the interpretation of these diverse phenomena in terms of driving physical mechanisms that have the same character.

1. Introduction

The new field of complex system studies holds that the dynamics of various complex systems are founded on universal principles, which can be used to describe disparate problems. A basic reason for our interest in complexity is the striking similarity in behavior near the global instability among systems that are otherwise quite different in nature [1–4]. A corollary is that transferring ideas and results from investigators in hitherto disparate areas will cross-fertilize and lead to important new results.

Mounting empirical evidence has been supporting the possibility that a number of systems arising in disciplines as diverse as physics, biology, engineering, and economics may have certain quantitative features that are intriguingly similar. These properties can be conveniently grouped under the headings of scale invariance and universality [1–6]. For instance, *de Arcangelis* et al. [6] have recently shown that the stochastic processes underlying apparently different phenomena such as solar flares and earthquakes have universal properties.

Nonextensive statistical mechanics provides a solid theoretical basis for describing and analyzing complex systems out of equilibrium. Boltzmann–Gibbs statistical mechanics works best in dealing with systems composed of subsystems which can access all the available phase space and which are either independent or interact via short-range forces. For systems exhibiting long-range correlations, memory, or fractal properties, nonextensive statistical mechanics through Tsallis entropy becomes the most appropriate mathematical tool [7–9]. In particular, Tsallis statistics is nowadays widely applied to solar

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and space physics problems such as heliosphere magnetic field and solar wind magnetic field [10,11], kappa distributions in space plasmas [12,13] and magnetosphere dynamics [14–16].

In Ref. [5] the similarities of multiple fracturing on a neutron star and on the Earth were explored, including power-law energy distributions based on the empirical relationship given by *Gutenberg and Richter* [17] for earthquakes. *Sornette and Helmstetter* [3] introduced a new kind of critical stochastic finite-time singularity and illustrated that this type of singularity occurs in epidemic models of rupture, earthquakes and starquakes associated with neutron stars. *de Arcangelis* et al. [6] showed that the same empirical laws widely accepted in seismology also characterize, surprisingly, the size and time occurrence of solar flares.

Universality relates to the uncovering of a universal formula [18,19] that describes with good approximation the same property on different systems. Here, we present evidence that the same energy distribution function deduced from an earthquake dynamics model developed in the context of nonextensive Tsallis statistics [20,21] also characterizes, surprisingly, the data of solar flares and magnetic storms. There are two major breakthroughs in our results. The observed similarity is not based on an empirical law but on an analytically derived formula that gives the Gutenberg–Richter law as a particular case. Moreover, this formula was deduced in the framework of the nonextensive Tsallis statistical theory, which can better describe nonequilibrium systems with large variabilities, such as solar corona, magnetosphere and lithosphere.

The magnetosphere and lithosphere are far from equilibrium because of the external driving by the turbulent solar wind [22] and a driving force arising from plate tectonic motions that increases stresses on the associated crust segment, respectively. For the case of solar corona, the generation of active regions is due to the emergence of magnetic flux (i.e., external driver) to the solar surface. Thus, solar corona, Earth's magnetosphere and lithosphere represent open (input–output) spatially extended nonequilibrium systems, which, on the one hand, are well organized in space and time, and, on the other hand, manifest their activity over many different spatial and temporal scales.

Solar flares are highly energetic explosions from active regions of the Sun in the form of electromagnetic radiation, particle acceleration and plasma flows powered by strong and twisted magnetic fields. It has been found that solar flares exhibit scale invariant statistics and that the probability distribution of flare energies is a power-law spanning more than 8 orders of magnitude, similar to the Gutenberg–Richter law for earthquakes [23–25]. Moreover, solar flares seem to be related with nonextensive concepts since their probability distributions of characteristic times appear to be of the q-exponential form [9,25].

Magnetic storms are the most prominent global phenomenon of geospace dynamics, interlinking the solar wind, magnetosphere, ionosphere, atmosphere and occasionally the Earth's surface. Magnetic storms are the main element of space weather: they have severe impacts on both space-borne and ground-based technological systems [26–28]. Therefore, their prediction has long been a goal of the space science community. The most widely used statistical descriptor of magnetic storm activity is the D_{st} index. This index is considered to reflect variations in the intensity of the symmetrical part of the magnetospheric ring current that circles Earth at altitudes ranging from about 3–8 Earth radii, and is proportional to the total energy in the drifting particles that form the ring current (http://swdcwww.kugi.kyoto-u.ac.jp/). It has been found that D_{st} exhibits a power-law behavior with the Hurst index varying over different intervals of the time series [29,30]. Moreover, *Balasis* et al. [14–16] analyzed D_{st} time series by introducing the nonextensive Tsallis entropy as an appropriate complexity measure to investigate the dynamics of the magnetosphere.

Sotolongo-Costa and Posadas [20] based on a nonextensive formulation of the maximum entropy principle analytically deduced an energy distribution function, which gives the Gutenberg–Richter law as a particular case. Their expression describes the energy distribution in all detectable ranges of earthquake magnitudes, very well, unlike the empirical formula of Gutenberg–Richter. Recently, *Silva* et al. [21] revised the formula introduced by *Sotolongo-Costa and Posadas* [20]. In Ref. [21] an energy distribution function is calculated analytically through the extremization of the Tsallis entropy under the constraints of the q-expectation value and the normalization condition.

2. Theoretical background

2.1. Principles of nonextensive Tsallis entropy

In nature, long-range spatial interactions or long-range memory effects may give rise to very interesting behaviors. Among them, one of the most intriguing arises in systems that are nonextensive (nonadditive). These systems share a very subtle property: they violate the Boltzmann–Gibbs statistics, the bridge to the equilibrium thermodynamics. Inspired by multifractal concepts, *Tsallis* [7,8] has proposed a generalization of the Boltzmann–Gibbs statistical mechanics. He introduced an entropic expression characterized by an index q which leads to nonextensive statistics, $S_q = k\frac{1}{q-1}\left(1-\sum_{i=1}^{W} p_i^q\right)$, where p_i are probabilities associated with the microscopic configurations, W is their total number, q is a real number and k is Boltzmann's constant. The entropic index q describes the deviation of Tsallis entropy from the standard Boltzmann–Gibbs entropy. Indeed, using $p_i^{(q-1)} = e^{(q-1)\ln(p_i)} \sim 1 + (q-1)\ln(p_i)$ in the limit $q \to 1$, we recover the usual Boltzmann–Gibbs entropy $S_{B-G} = -k\sum_{i=1}^{W} p_i \ln(p_i)$.

The nonextensive formulation seems to present a consistent theoretical tool to investigate complex systems in their nonequilibrium stationary states, systems with multifractal and self-similar structures, systems dominated by long-range

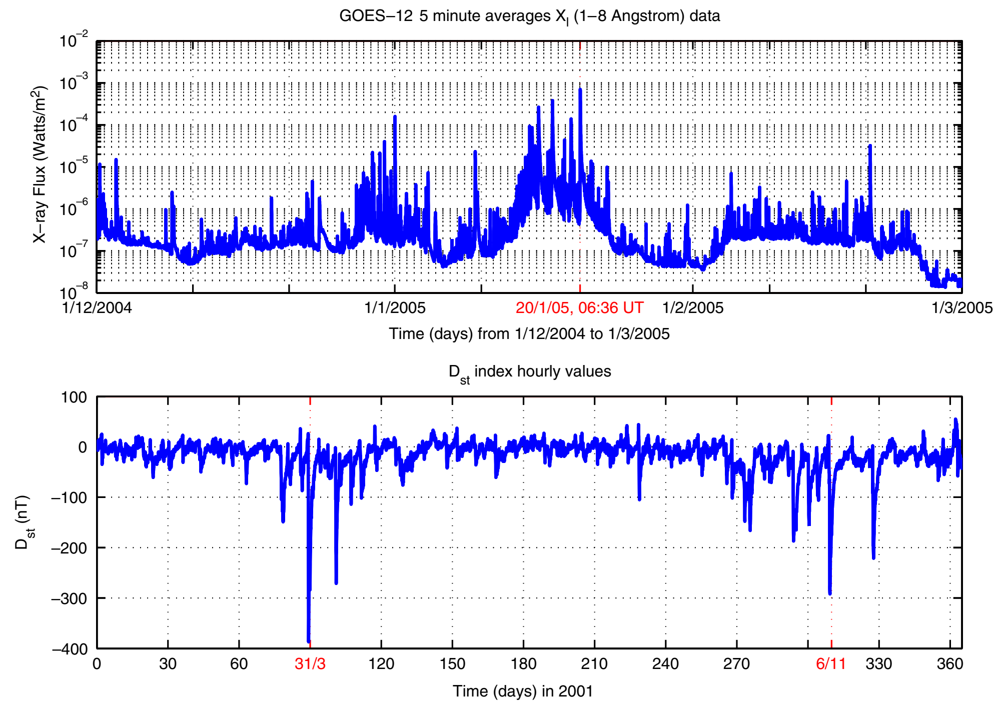

Fig. 1. X-ray flux time series (upper panel). The 20 January 2005 solar flare is marked with red. D_{st} time series (lower panel). The 31 March and 6 November 2001 magnetic storms are marked with red.

interactions, and anomalous phenomena among others. The entropic index q characterizes the degree of nonextensivity reflected in the following pseudo-additivity rule $S_q(A+B) = S_q(A) + S_q(B) + \frac{1-q}{k} S_q(A) S_q(B)$.

For subsystems that have special probability correlations, extensivity $S_{B-G} = S_{B-G}(A) + S_{B-G}(B)$ is not valid for S_{B-G}, but may occur for S_q with a particular value of the index q. Such systems are sometimes referred to as nonextensive [7,8]. The cases $q > 1$ and $q < 1$, correspond to sub-additivity, or super-additivity, respectively. We may think of q as a bias-parameter: $q < 1$ privileges rare events, while $q > 1$ privileges prominent events.

We clarify that the parameter q itself is not a measure of the complexity of the system but measures the degree of nonextensivity of the system. It is the time variations of the Tsallis entropy for a given $q(S_q)$ that quantify the dynamic changes of the complexity of the system. Lower S_q values characterize the portions of the signal with lower complexity.

2.2. The energy distribution function

A model for earthquake dynamics coming from a nonextensive Tsallis formalism, starting from first principles, has been recently introduced by *Sotolongo-Costa and Posadas* [20]. This approach leads to an energy distribution function (Gutenberg–Richter type law) for the magnitude distribution of earthquakes (see Eq. 8 in Ref. [20]). Their equation provides an excellent fit to seismicities generated in various large geographic areas usually identified as seismic regions. *Silva* et al. [21] have subsequently revised this model considering the current definition of the mean value, i.e., the so-called q-expectation value. They also suggested an energy distribution function, which provides an excellent fit to seismicities, too

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right) \log\left[1 - \left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{\alpha^{2/3}}\right)\right], \quad (1)$$

where N is the total number of earthquakes, $N_{>m}$ the number of earthquakes with magnitude larger than m, and $m \approx \log \varepsilon$. α is the constant of proportionality between the earthquake energy, ε, and the size of fragment, r ($\varepsilon \sim r^3$). Importantly, the associated q-values with the aforementioned Gutenberg–Richter type law (Eq. (1)) for 3 different regions (faults) in the world and by considering a threshold (m) equal to earthquake magnitude 3 are 1.6, 1.63 and 1.71, respectively.

3. Universality in solar flares, magnetic storms and earthquakes

The X-ray flux data used in this study include a series of M- and X-class solar flares occurred in the single extensive active region AR0720 between 10 and 23 January 2005 (see Fig. 1 upper panel). In particular, between January 15th and 19th, this sunspot produced four powerful solar flares. When it exploded a fifth time on January 20th released the highest

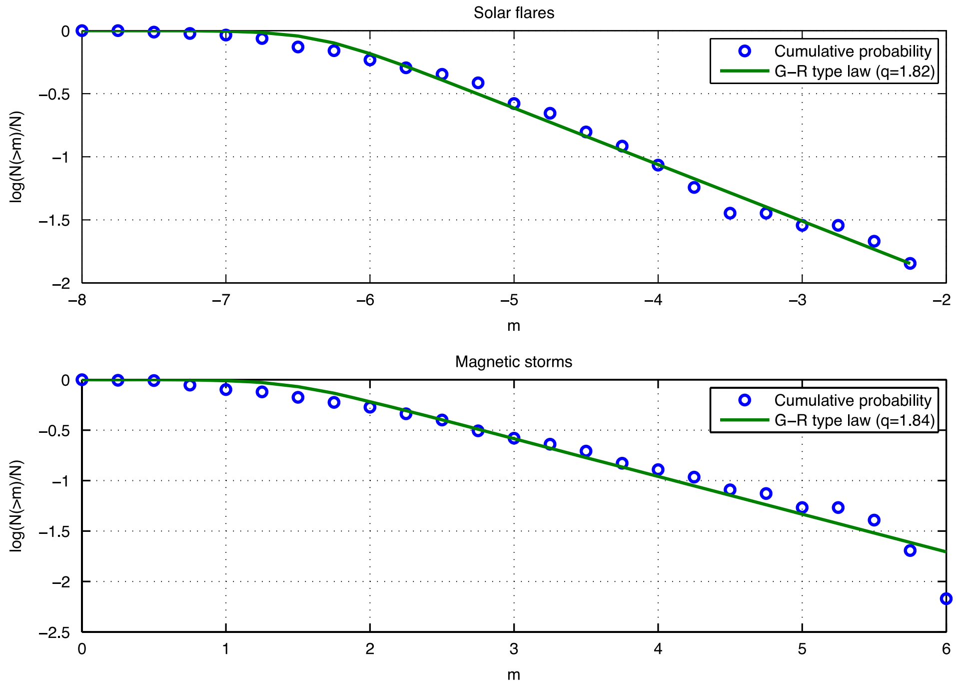

Fig. 2. We use the Gutenberg–Richter (G–R) type law for the nonextensive Tsallis statistics (Eq. (1)) to calculate the relative cumulative number of X-ray flux data, $N_{>m}/N$ (upper panel). There is an excellent agreement of the aforementioned formula with the X-ray flux time series. The threshold is 10^{-6} W/m^2 which results in 141 events, and the associated parameter is $q = 1.82$. We then use Eq. (1) to calculate the relative cumulative number of D_{st} data, $N_{>m}/N$ (lower panel). There is an excellent agreement of the aforementioned formula with the D_{st} time series. The threshold is −30 nT which results in 164 events, and the associated parameter is $q = 1.84$.

concentration of protons ever directly measured, taking only 15 minutes after observation to reach Earth, indicating a velocity of approximately one-half light speed. The D_{st} data utilized here include two intense magnetic storms, which occurred on 31 March 2001 and 6 November 2001 with minima D_{st} −387 nT and −292 nT, respectively, as well as a number of smaller events (e.g. May and August 2001 with $D_{st} \sim -100$ nT in both cases) (see Fig. 1 lower panel). Next, we simultaneously describe our results for the cases of solar flares and magnetic storms.

We now examine whether the energy distribution function (Eq. (1)) corresponding to a nonextensive Tsallis statistics is able to describe the X-ray flux (D_{st}) time series. Fig. 2 upper (lower) panel shows that Eq. (1) provides an excellent fit to the experimental data, incorporating the characteristics of nonextensivity statistics into the distribution of the solar (magnetospheric) events. Herein, N is the total number of X-ray flux (D_{st}) data, $N_{>m}$ the number of X-ray flux (D_{st}) values with magnitude larger than m, $G_{>m} = N_{>m}/N$ the relative cumulative number of events with magnitude larger than m, and α a proportionality constant. The magnitude m is approximately $\log \varepsilon$, where ε is the integrated X-ray flux (squared D_{st}) value. (In the case of X-ray flux we take the integral of fluxes for calculating energy, whereas in the case of D_{st} index the square of the amplitude of the magnetic field is proportional to energy). The best-fit for this analysis is given by a q parameter value equal (within a confidence limit of 95%) to 1.82 (1.84), whereas the threshold m is taken 10^{-6} W/m^2 (−30 nT). (The parameters associated with the model minimize the χ^2 merit function which is given by the sum of the squared residuals.)

For the case of D_{st} index, the energy associated with each data point is given by the square of the difference between the field value and a background noise level (threshold). For the case of X-ray data, where the measured quantity is the energy flux, there is no need to use the square of the data. In both cases, we consider a sequence of N consecutive values that surpass the threshold to constitute an "event" and measure its energy by integrating (summing) the individual data-point energies that comprise it. Though this method better captures the system dynamics, it tends to unify near-concurrent events when they both exceed the corresponding threshold, thus attributing lower probabilities to very energetic events. This "merging" explains the divergence from the theoretical curve at the far right of the graph in the lower panel of Fig. 2.

We note that the estimated nonextensive q parameter values are in full agreement with the upper limit $q < 2$ obtained from several studies involving the Tsallis nonextensive framework [31,32]. Moreover, it is in harmony with an underlying sub-extensive system, $q > 1$, verifying the emergence of strong interactions in the solar corona and Earth's magnetosphere during the preparation process of solar flares and magnetic storms, respectively.

The aforementioned result indicates that solar flares, magnetic storms and earthquakes obey nonextensive laws which are scale invariant, and that these laws are universal in the sense that they do not depend on details concerning the actual species. The aforementioned finding could be considered as a further indication of the universality of fractal properties among a large number of various geophysical processes.

4. Discussion and conclusions

Herein, the principle of universality in solar flare, magnetic storm and earthquake dynamics is established. The aforementioned similarity is quantitatively supported by the observation of power-law in the distribution of solar flare and magnetic storm energy related to a nonextensive Tsallis formalism that gives the Gutenberg–Richter law for the earthquake magnitude distribution as a special case.

The observed universal dynamics in solar flares, magnetic storms and earthquakes, on the basis of a nonextensive Tsallis energy distribution function with similar q indices (i.e., 1.82 for flares, 1.84 for storms and 1.6–1.71 for earthquakes), suggests a common approach to the interpretation of these phenomena in terms of driving physical mechanisms that have the same character. For instance, *de Arcangelis* et al. [6] suggested that magnetic stress transfer in the solar corona plays the role of elastic stress redistribution on the Earth's crust. On the other hand, plasma pressure distribution in the inner magnetosphere is one of the key parameters for understanding the development of magnetic storms. Recently, *Tsyganenko* [33] demonstrated a dramatic increase of the plasma pressure profiles from quiet to disturbed geomagnetic conditions. Therefore, plasma pressure redistribution in the Earth's magnetosphere could play the role of magnetic stress transfer in the solar corona and elastic stress redistribution on the Earth's crust.

Solar corona, Earth's magnetosphere and lithosphere can be considered as externally driven input–output systems. For the case of solar corona, the generation of active regions is due to the emergence of magnetic flux (i.e., external driver) to the solar surface. The occurrence of solar flares can be considered as a relaxation process, related to the sudden energy release of the accumulative free magnetic energy, when a critical value of magnetic field is reached. Similarly the Earth's magnetosphere is a system that it is continuously driven externally by the solar wind velocity and magnetic field. Storm is an interval of time when a sufficiently intense and long-lasting interplanetary convection electric field leads, through a substantial energization in the magnetosphere–ionosphere system, to an intensified ring current strong enough to exceed some key threshold of the quantifying storm time D_{st} index [34]. Lithospheric stress, on the other hand, is increased when tectonic plates move against each other. When the stress is large enough, the crust is forced to break. Furthermore, the observation that solar flares and magnetic storms have almost identical q values can be attributed to the fact that both solar flares and magnetic storms are driven by solar activity.

Recently, De Freitas and De Medeiros [35] used a new approach to study the nonextensivity properties of solar magnetic activity from 1996–2001. The study was carried out on daily measurements of Sunspot Numbers, mean magnetic-field strength, and daily means of Total Solar Irradiance. Probability Distribution Functions were calculated for the three datasets and the obtained results showed that the recently proposed by Tsallis entropic indices (also known as q-Triplet [36]) change as a function of scale. Moreover, Carvalho et al. [37] applied both Tsallis and Kaniadakis statistics [38,39] to the puzzling astrophysical problem of the function governing the distribution of stellar rotational velocity. They have shown for the first time that these are by far the most appropriate statistics for this problem giving a very good fit. It would be of course interesting and worthwhile to explore those methodologies in the case of solar flares, magnetic storms and earthquakes at subsequent work.

The evidence of a universal statistical behavior suggests the possibility of a common approach to forecasting of space weather and earthquakes. In any case, the transfer of ideas and methods of seismic forecasting to the prediction of solar flares and magnetic storms could improve space weather forecasting.

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